Seat No:________________ Enrollment No:____________________

# C.U.SHAH UNIVERSITY

**WADHWAN CITY**

**University (Winter) Examination -2013**

***Name :M.Sc(Mathematics) Sem-I*** **Subject Name: -Complex Analysis -I** **Marks :70**

**Duration :- 3:00 Hours** **Date : 20/12/2013**

**Instructions:-**

**(1)** Attempt all Questions of both sections in same answer book / Supplementary.
**(2)** Use of Programmable calculator & any other electronic instrument is prohibited.
**(3)** Instructions written on main answer Book are strictly to be obeyed.
**(4)**Draw neat diagrams & figures (If necessary) at right places.
**(5)** Assume suitable & Perfect data if needed.

## SECTION-I

Q-1 a) Find the value of $i^{100}$. (01)

b) Find imaginary value of $z = \frac{5}{(1-i)(1+i)}$. (01)

c) $w = \log z$ is analytic everywhere except at $z =$ _____. (01)

d) Evaluate: $\int_i^1 (z+1)^2 \; dz$. (01)

e) Evaluate: $\int_C \frac{1}{z^3} \; dz, \; C: |z| = 1$. (01)

f) Let $C$ be a circle $|z-1| = 3$ in the complex plane, then find $\int_C \frac{\cos z}{z-\pi} \; dz$. (01)

g) $\oint_C \frac{z+1}{z^2} \; dz = 2\pi i$, Where $C$ is a path enclosing the origin. Determine whether the statement is true or false. (01)

Q-2 a) Suppose a function $f(z) = u(x,y) + iv(x,y)$ is defined in the neighborhood of a point $z_0 = x_0 + iy_0$. Then prove that $f$ is differentiable at $z_0$ if (05)
i) C-R equations are true at $z_0$ and
ii) First order partial derivatives $u_x, u_y, v_x$ and $v_y$ are continuous at $z_0$.

b) Find out the complex number $\left(\frac{1+\sin\alpha + i\cos\alpha}{1+\sin\alpha - i\cos\alpha}\right)^n$. (05)

c) Show that if $C$ is any $n^{th}$ root of unity other than unity itself, then prove that $1 + C + C^2 + \cdots + C^{n-1} = 0$. (04)

**OR**

Q-2 a) Define: Conjugate of complex number. Show that $z^2 = \bar{z}^2$ if and only if $z$ either real or purely imaginary. (05)

b) Write polar form of C-R equation. Using it $f(z) = z^{\frac{5}{2}}$ is analytic or not. (05)

c) Is $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$? Justify. (04)

Q-3 a) Prove that $f(z) = u + iv$ is analytic on a domain $D$ if and only if $v$ is a harmonic conjugate of $u$ on $D$. (05)

b) Let $f(z)$ be analytic on a domain $D$ with fact that $f'(z) = 0$ for all $z$ in the domain $D$. Then prove that $f$ reduces to a constant on $D$. (05)

c) Show that $\lim_{z \to 2i} (2x + iy^2) = 4i$. (04)

**OR**

Q-3 a) Find out the analytic function having the real part is $u(x, y) = y + e^x \cos y$. (05)

b) If $f(z)$ is analytic within and on a simple close curve $C$ and $z_0$ is any point interior to $C$, then prove that $\oint_C \frac{f(z)}{z-z_0}\, dz = 2\pi i\, f(z_0)$ the integration being taken counterclockwise. (05)

c) State and prove ML- inequality. (04)

## SECTION-II

Q-4 a) Find Laurent series of $f(z) = z^2\, e^{\frac{1}{z}}$ about the indicated point $z_0 = 0$. (02)

b) Find pole of $f(z) = \frac{1}{(z+i)}$. (01)

c) Using residue theorem, show that $\oint_C \frac{e^{-z}}{z^2}\, dz = -2\pi i$, $C$ is the circle about the origin. (01)

d) State maximum modulus principle. (01)

e) Determine orders of zeros of $f(z) = z \sin\left(\frac{1}{z}\right)$. (01)

f) State Rouche's Theorem. (01)

Q-5 a) State Laurent Expansion Theorem. Find the Laurent expansion of $f(z) = \frac{1}{z^2 - z}$ near $z = 0$. (05)

b) Define: Bilinear Transformation. Find the bilinear transformation that maps the points $z = 0, 1, \infty$ in to the points $w = -5, -1, 3$ respectively. (05)

c) Find radii of convergence of power series $i)\ \sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$, $\quad ii)\ \sum_{n=1}^{\infty} \frac{1}{n^p} z^n$. (04)

**OR**

Q-5 a) Let $C_1$ be a positively oriented simple close contour and $C_2$ be another simple closed contour which is negatively oriented and lying in $C_1$. Suppose $f$ is analytic on the closed region bounded by $C_1$ and $C_2$, then prove that $\int_{C_1} f(z)\, dz + \int_{C_2} f(z)\, dz = 0$. (05)

b) Write Maclaurin's Series. Find the Maclaurin series representation of $f(z) = \sin z$ in the region $|z| < \infty$. (05)

c) Find the fixed points of the transformation $w = \frac{-2+(2+i)z}{i+z}$. (04)

Q-6 a) Let $C$ be a simple closed contour, described in the positive sense. If a function $f(z)$ is analytic inside and on $C$ except for a finite number of singular points(poles or isolated singularities) $z_1, z_2, \dots, z_n$ inside $C$, then prove that $\oint_C f(z)\, dz = 2\pi i \sum_{k=1}^{n} \underset{z = z_0}{Res} f(z)$. (05)

b) Define residue at simple pole and find the sum of residues of the function $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$. (05)

c) Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about $z = 0$ and identify the singularity. (04)

Q-6 a) Evaluate : $\int_0^{2\pi} \frac{4\, d\theta}{5+4\sin\theta}$. (05)

b) Define Residue at multipole. Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and find the residue at each pole. (05)

c) Define Isolated, Essential and Removable singularities with examples. (04)

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